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Technical Report 581

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A Comparison of the SAOC with the AGK3 Hemispherical Expansions

21 October 1981

Prepared for the Department of the Air Force
under Electronic Systems Division Contract F19628-80-C-0002 by

Lincoln Laboratory

MASSACHUSETTS INSTITUTE OF TECHNOLOGY

LEXINGTON, MASSACHUSETTS

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The work reported in this document was performed at Lincoln Laboratory, a center for research operated by Massachusetts Institute of Technology, with the support of the Department of the Air Force under Contract F19628-80-C-0002.

This technical report has been reviewed and is approved for publication.

FOR THE COMMANDER

Raymond L. Loiselle

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LINCOLN LABORATORY

A COMPARISON OF THE SAOC WITH THE AGK3 HEMISPHERICAL EXPANSIONS

L.G. TAFF

Group 94

TECHNICAL REPORT 581

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ABSTRACT

This is the second report on the systematic differences between the AGK3 and SAOC positions and proper motions. The general theory of formulating such differences, in three different circumstances, is developed and the method of (hemi)spherical harmonics applied. Magnitude and color terms are treated, through terms quadratic in m and c, by extending the variable space via tensor product spaces. The statistical significance of an expansion coefficient is ascertained by the F test. Also included is a discussion of the time dependence of these differences and a resolution of most of the poorly matching stars (Table 2 of Part I).

CONTENTS

I. INTRODUCTION

In an earlier Report[1] we presented the classical method of analyzing the positions and proper motions of stars in common to two star catalogs. It was applied to the Smithsonian Astrophysical Observatory Star Catalog[2] (the SAOC hereinafter) and the third of the Catalog der Astronomischen Gesellschaft[3] (the AGK3 hereinafter). The motivation and importance of such work was given there too. This Report explores the method of using a complete set of orthonormal basis functions to perform the same task. Brosche[4,5] was the first to propose and do this in astronomy. Moreoever, following the public presentation of the first part of reference 1[6], we received two suggestions for further work. Dr. Clayton A. Smith (U.S.N.O.) advised us to look into the time dependence of the differences between the AGK3 and the SAOC. This is presented in Appendix II. Dr. John M. Sorvari (LL-M.I.T.) proposed some astrophysical and observational selection effects which might explain the results of the magnitude and color effects found earlier. Our discussions and ideas on these points are in Appendix III.

The next section describes a general procedure for expanding any appropriately smooth function of right ascension and declination over an arbitary "rectangular" region on a spherical surface. (All questions of convergence and smoothness are intentionally avoided. The theory of generalized Fourier expansions is well developed and we can't definitely answer convergence questions

from numerical data.) We rapidly specialize to hemispherical harmonics for the northern celestial sphere since the AGK3 only extends southward to $\delta = -2^\circ\!.5$. The incorporation of magnitude and color effects occupies our attention next. All of this is especially easy to implement computationally when the parameter distribution (i.e., α, δ, m, and c) is both uniform and dense. The theoretical basis for the analysis, in two different ways, is developed next. Sections IIID and IIIE may be skipped, therefore, without loss.

The last section presents the results for the positions and proper motions with and without the magnitude and color terms. Appendix I is a full FORTRAN source listing of our $P_{nm}(x)$ generating subroutine. It's an adaptation of the subroutine LEGPOL from the University of Rochester Computing Center (#309.2.507). The programming for the majority of this Report was performed by Sharon A. Stansfield.

II. EXPANSIONS OVER "RECTANGULAR" REGIONS ON A SPHERICAL SURFACE

By "rectangular" region we mean any region of a spherical surface that can be described as consisting of the simply connected interior to $\alpha = \alpha_L$, $\alpha = \alpha_U$ and $\delta = \delta_L$, $\delta = \delta_U$. We also have $0 \leq \alpha_L \leq \alpha_U \leq 2\pi$, $-\pi/2 \leq \delta_L \leq \delta_U \leq \pi/2$ and we don't pay attention to the strict inclusion or exclusion at any boundary. This definition will allow us to handle all star catalogs of interest.

A. In General

Define $Y_{nm}(\alpha,\delta)$, the usual spherical harmonic, by

$$Y_{nm}(\alpha,\delta) = N_{nm} P_{nm}(\sin\delta)\exp(im\alpha) \tag{1a}$$

where

$$N_{nm} = \left[\frac{2n+1}{4\pi} \frac{(n-m)!}{(n+m)!}\right]^{1/2} \tag{1b}$$

and P_{nm} is the associated Legendre function of order n, degree m;

$$P_{nm}(x) = \frac{(-1)^m}{2^n n!}(1-x^2)^{m/2} \frac{d^{n+m}}{dx^{n+m}}(x^2-1)^n$$

$$= (-1)^m (1-x^2)^{m/2} \frac{d^m P_n(x)}{dx^m} \tag{1c}$$

Here $P_n(x) \equiv P_{no}(x)$ is the ordinary Legendre polynomial of order n. The set $\left\{Y_{nm}(\alpha,\delta)\right\}$ $n = 0,1,2,\ldots;\ m = -n, -n+1,\ldots,n$ forms a complete set of orthonormal basis functions on a spherical surface, viz.

$$\sum_{n=o}^{\infty} \sum_{m=-n}^{+n} Y_{nm}(\alpha,\delta)Y^{*}_{nm}(\alpha',\delta') = \delta(\alpha-\alpha')\delta(\sin\delta-\sin\delta') \tag{2a}$$

$$\int_{o}^{2\pi} d\alpha \int_{-\pi/2}^{\pi/2} Y_{nm}(\alpha,\delta)Y^{*}_{n'm'}(\alpha,\delta)\cos\delta d\delta = \delta_{nn'}\delta_{mm'} \tag{2b}$$

where the asterisk denotes complex conjugation, $\delta(u)$ is the Dirac delta function, and δ_{pq} is the Kronecker delta function.

For any (see comment on page 1) function $f(\alpha,\delta)$ we can write

$$f(\alpha,\delta) = \sum_{n=o}^{\infty} \sum_{m=-n}^{+n} f_{nm}Y_{nm}(\alpha,\delta) \tag{3a}$$

where

$$f_{nm} = \int_{o}^{2\pi} d\alpha \int_{-\pi/2}^{\pi/2} f(\alpha,\delta)Y^{*}_{nm}(\alpha,\delta)\cos\delta d\delta \tag{3b}$$

Now define $y_{nm}(\alpha,\delta)$ by

$$y_{nm}(\alpha,\delta) = \left[\frac{2}{(\alpha_U-\alpha_L)\ (\sin\delta_U-\sin\delta_L)}\right]^{1/2} N_{nm} \cdot$$

$$P_{nm}\left[\frac{2\sin\delta-(\sin\delta_L+\sin\delta_U)}{\sin\delta_U-\sin\delta_L}\right]\cdot \tag{4}$$

$$\exp\left[2\pi i m(\alpha-\alpha_L)/(\alpha_U-\alpha_L)\right]$$

Then it follows from the properties of the $\{Y_{nm}\}$ that for any function $f(\alpha,\delta)$ defined on the "rectangle" $\alpha\varepsilon[\alpha_L,\alpha_U]$, $\delta\varepsilon[\delta_L,\delta_U]$ (with any square bracket replaceable by a parenthesis) we can write

$$f(\alpha,\delta) = \sum_{n=o}^{\infty}\ \sum_{m=-n}^{+n}\ f_{nm}y_{nm}(\alpha,\delta) \tag{5a}$$

with

$$f_{nm} = \int_{\alpha_L}^{\alpha_U} d\alpha \int_{\delta_L}^{\delta_U} f(\alpha,\delta)y^*_{nm}(\alpha,\delta)\cos\delta d\delta \tag{5b}$$

B. Hemispherical Harmonics

Because the AGK3 is a Northern hemisphere catalog, we're primarily interested in Northern hemispherical harmonics. The Southern extension is straightforward.

Define $H_{nm}(\alpha,\delta)$ by

$$H_{nm}(\alpha,\delta) = \sqrt{2}\, N_{nm} P_{nm}(2\sin\delta - 1)\exp(im\alpha) \tag{6}$$

Then the $\{H_{nm}\}$ forms a complete set of orthonormal basis functions for the Northern half of the celestial sphere. We note in passing that only the combinations considered above, Eqs. (4,6), have the requisite properties of being both complete orthonormal basis functions in their domain of definition and require no ad hoc presumptions concerning the nature of $f(\alpha,\delta)$ in regions outside of the rectangle. Hence, the proposal of Bien et. al.[7] [i.e., their Eq. (17) wherein a Fourier series in right ascension <u>decoupled</u> from a Legendre <u>polynomial</u> expansion in declination is put forward] is somewhat curious. Contrarily (cf. section IIC) their use of Hermite polynomials to accommodate the magnitude term appears to be overkill.

C. Magnitude and Color Effects

The functions $f(\alpha,\delta)$ we shall deal with will be one of

$$\begin{aligned}
\Delta\alpha\cos\delta &\equiv \left(\alpha_{AGK3} - \alpha_{SAOC}\right)\cos\delta_{SAOC} \\
\Delta\delta &\equiv \delta_{AGK3} - \delta_{SAOC} \\
\Delta p &\equiv \left[(\Delta\alpha\cos\delta)^2 + (\Delta\delta)^2\right]^{1/2}
\end{aligned} \tag{7}$$

or one of $\Delta\mu_\alpha \cos\delta$, $\Delta\mu_\delta$, or $\Delta\mu = \left[(\Delta\mu_\alpha \cos\delta)^2 + (\Delta\mu_\delta)^2\right]^{1/2}$ similarly defined. Not only will these quantities depend on position, they may also depend on the brightness and temperature of the stars. Thus we would write for (say) $\Delta\delta$, $\Delta\delta(\alpha,\delta,m,c)$. To incorporate this in Eq. (6), without loss of generality, we write

$$\Delta\delta_{mn}(m,c) = \sum_{r=o}^{\infty} \sum_{s=o}^{\infty} \left[\frac{4}{(m_U - m_L)\ (c_U - c_L)}\right]^{1/2} \Delta\delta_{mn}^{rs} \cdot$$

$$P_r\left[\frac{2m-(m_L+m_U)}{m_U-m_L}\right] \cdot P_s\left[\frac{2c-(c_L+c_U)}{c_U-c_L}\right] \tag{8}$$

for $m\varepsilon[m_L, m_U]$ and $c\varepsilon[c_L, c_U]$.

III. NUMERICAL CONSIDERATIONS

A. In General

It is clear that in any numerical implementation of the above one will truncate the series at some upper limit, say $n\epsilon[0,N]$. A standard result of analysis is that the best, in the least squares sense, fit of some function f(x) to an arbitrary orthonormal basis $\{p_n\}$ on $x\epsilon[a,b]$ to order N, viz.

$$f(x) \sim \sum_{n=0}^{N} f_n p_n(x) \tag{9a}$$

occurs when the expansion coefficients f_n are given by

$$f_n = \int_a^b f(x) p_n^*(x) dx. \tag{9b}$$

To be explicit, the representation (9b) minimizes

$$R_N = \int_a^b \left| f(x) - \sum_{n=0}^{N} f_n p_n(x) \right|^2 dx \tag{9c}$$

Given this, it would appear that we're left with a numerical quadrature problem as the f_n in Eq. (9b) are independent. This isn't quite true. As long as N isn't infinite there's the possibility that some part of f isn't being modeled. This is an incompleteness problem and will be reflected by the fact that R_N is still larger than the variance due to accidental (as opposed to systematic) effects. The independence of the

$\{f_n\}$ does assure us, though, that the unmodeled component of f isn't being projected elsewhere; it simply will be left out.

Having disposed of that point we are left with a numerical quadrature problem. When the data densely spans the range of x any scheme that will accurately integrate a 2N'th order polynomial will do. Note though, that the f_n will not necessarily be stable as a function of the order of the quadrature scheme because as we go to higher order (which also means higher resolution in x) we may be discovering finer details in f(x).

If the data isn't uniformly dense or the value of N isn't a priori decided, then the problem becomes more complex. In the first case the sparseness of the data means that a numerical implementation of Eq. (9b) may destroy the orthogonality property. With this occurring while we vary the value of N the independence of the $\{f_n\}$ vanishes. This can be handled with a Gram-Schmidt orthogonalization procedure (section IIID) as N varies or by ordinary least squares (section IIIE) with N fixed. The reader may want to skip these two tangential sections on a first reading. Below we detail the numerical procedures employed here. Almost all of the statistical questions relevant to this problem one might pose are sloughed off here because we feel that they're immaterial.

B. Hemispherical Harmonics

The values of $P_{nm}(x)$ were computed using the code in Appendix I. The values obtained were checked against reference 8. $P_{nm}(x)$ was computed using 64 bit arithmetic and then truncated to 32 bits. Gauss-Legendre quadrature schemes using 16 and 32 base points were employed to perform the declination integrals in Eq. (5b) with $\delta_L = 0$, $\delta_U = 90°$. We judged that the analytical simplicity of hemispherical harmonics outweighed the 4.4% loss engendered $\left\{(2^{\circ}\!.5 \times 360°)\ [4\pi(180°/\pi)^2] = 0.022\right\}$. This still leaves us with 130,000 stars nearly uniformly distributed over the Northern half of the celestial sphere. The base points in each case are the zeroes of $P_L(x)$, L = 16 or 32. Their mean separation is $5^{\circ}\!.3 \pm 2^{\circ}\!.3$ and $2^{\circ}\!.7 \pm 1^{\circ}\!.2$ and if $P_L(x_\ell) = 0$ then $\delta_\ell = \sin^{-1}[(1 + x_\ell)/2]$. From Eq. (1c) we see that if N is fixed at 15 (the value we've used) that the highest order "polynomial" we need to handle is 30. The normalization integrals for the $\left\{H_{nm}\right\}$, $n = 0,1,2,\ldots,15$, $m = -n, -n + 1,\ldots,n$ were performed as a check and never differed from unity by more than 5×10^{-5}. Newton-Cotes type formulas, even at $\Delta\delta = 0.01$, were not accurate. We also would prefer not to prejudice the results by having different weights at different positions but have no choice.

The closeness of the mean separations in declination, plus our just mentioned bias, led us to the trapezoidal rule at $\Delta\alpha = 5°$ and $2^{\circ}\!.5$ (as appropriate) to perform the right ascension

integral ($\alpha_L = 0$, $\alpha_U = 360°$) in the hemispherical harmonic version of Eq. (5b). Again the normalization integrals were used to check the accuracy. The worst deviation from unity was 0.014 at 5° and 0.007 at 2°5. The possibility of using a Filon type formula was also investigated.

The details of forming (say) $\Delta\delta(\alpha,\delta)$ for the numerical integrations were as follows: Fix L = 16 or 32 and $\Delta\alpha = 5°$ or 2°5 correspondingly. Compute the $\{\delta_\ell\}$ and form the midpoints of each bin (more complicated splittings have no better theoretical basis and don't move the boundaries significantly). We now have a grid over the Northern half of the celestial sphere. Differences for each star are placed in the appropriate elements of an array and the average value of each difference separately computed for each bin. These are the numbers used for (say) $\Delta\delta(\alpha,\delta)$. In addition to computing the mean we also calculated the variance in each bin (with respect to the mean) and then used these to provide an estimate of the variance of an individual value of $\Delta\delta_{nm}$.

C. Magnitude and Color Effects

Formally the problem is as above. Practically one must insure uniformity and density. As we have photographic magnitudes for all of the stars (from the AGK3), as long as we stay within the bulk of the magnitude distribution and don't use too fine an areal grid there'll be no difficulty. From Table 9

of reference 1 we choose, therefore, $m_L = 8^m.0$ and $m_U = 11^m.0$ and don't look for more than a quadratic term in m. For the color indices the situation is not as good. We again restrict ourselves to at most terms in c^2 and use $c_L = 0^m.0$, $c_U = 2^m.0$. A 4 point Gauss-Legendre quadrature scheme was used in each case and to minimize potential difficulties with sparseness the 16 x 5° scheme was used.

D. Gram-Schmidt Orthogonalization

A standard result of linear algebra is that if one has a set of K linearly independent vectors $\{\underline{u}_k\}$ k = 1,2,...,K then there exists a set of orthonormal vectors $\{\underline{e}_k\}$ that spans the same space. The $\{\underline{e}_k\}$ are defined through the intermediaries $\{\underline{v}_k\}$;

$$\underline{v}_1 = \underline{u}_1$$

$$\underline{v}_2 = \underline{u}_2 - \left(\frac{\underline{u}_2 \cdot \underline{v}_1}{|\underline{v}_1|^2}\right) \underline{v}_1$$

$$\underline{v}_3 = \underline{u}_3 - \left(\frac{\underline{u}_3 \cdot \underline{v}_2}{|\underline{v}_2|^2}\right) \underline{v}_2 - \left(\frac{\underline{u}_3 \cdot \underline{v}_1}{|\underline{v}_1|^2}\right) \underline{v}_1 \tag{10}$$

$$\underline{v}_k = \underline{u}_k - \sum_{\ell=1}^{k-1} \left(\frac{\underline{u}_k \cdot \underline{v}_\ell}{|\underline{v}_\ell|^2}\right) \underline{v}_\ell$$

$$\left\{\underline{e}_k = \underline{v}_k / |\underline{v}_k|\right\}$$

This construction is known as the Gram-Schmidt orthogonalization process, the normalization occurring in the last step.

Consider Eqs. (9) when N is variable and the values of f(x) are not dense on [a,b]. Then, since the practical version of Eq. (9a) is

$$f(x_\ell) = \sum_{n=o}^{N} f_n p_n(x_\ell) + \varepsilon_\ell, \quad \ell = 1,2,\ldots,L \tag{11}$$

for L samples of f where ε_ℓ is the noise, we can't simply obtain f_n via the extension of (9b),

$$f_n = \sum_{\ell=1}^{L} f(x_\ell)\, p_n^*(x_\ell)$$

Brosche[4] when dealing with this situation suggested applying the Gram-Schmidt procedure to the $\{p_n(x_\ell)\}$. In particular set $\underline{p}_n = [p_n(x_1), p_n(x_2), \ldots, p_n(x_L)]$ and define $\{\underline{q}_n\}$ via

$$\underline{q}_o = \underline{p}_o$$

$$\underline{q}_n = \underline{p}_n - \sum_{m=o}^{n-1} a_{nm}\underline{q}_m, \quad a_{nm} = \frac{\underline{q}_m \cdot \underline{p}_n}{|\underline{q}_m|^2} \tag{12a}$$

then, with $a_{kk} = 1$ and $A = \{a_{nm}\}$, $p = \{\underline{p}_n\}$, $q = \{q_n\}$

$$\underline{p}_n = \sum_{m=o}^{n} a_{nm}\underline{q}_n \quad \text{or} \quad p = Aq \tag{12b}$$

The matrix A is triangular and non-singular (because the linearly independent $\{\underline{p}_n\}$ span the vector space). The inverse is given by

$$\underline{q}_n = \sum_{m=o}^{n} b_{nm}\underline{p}_m \tag{13a}$$

where

$$b_{kk} = 1, \; b_{nm} = - \sum_{\ell=m}^{n-1} a_{n\ell} b_{\ell m} \text{ for } m < n \tag{13b}$$

and AB = I, the N dimensional unit matrix. Equation (11) is transformed into

$$\underline{f} = \sum_{n=o}^{N} F_n \underline{q}_n + \underline{E} \tag{14a}$$

where $\underline{f} = \left[f(x_1), f(x_2), \ldots, f(x_L)\right]$, $\underline{E}$ is the new noise vector, and $\{F_n\}$ are the expansion coefficient of $\underline{f}$ in the $\underline{q}$ basis. As the $\{\underline{q}_n\}$ are mutually orthogonal the least squares computation of $\underline{F}$ is trivial,

$$F_n = \frac{\underline{q}_n \cdot \underline{f}}{|\underline{q}_n|^2} \tag{14b}$$

and the original set of $\{f_n\}$ are given by

$$f_n = \sum_{m=n}^{N} F_m b_{mn} \tag{14c}$$

This completes the solution of the original problem taking into account the sparseness of $\left\{f(x_\ell)\right\}$ and a variable N. Brosche[4] goes on to discuss methods of determining the best value of N and assessing the significance of the $\left\{f_n\right\}$. Note that no large matrices need be inverted.

E. Least Squares

We consider the problem of representing $f(\alpha,\delta;m)$ in the form

$$f(\alpha,\delta;m) = \sum_{r=o}^{R} \sum_{p=o}^{R} \sum_{q=-p}^{+p} f_{pqr} Y_{pq}(\alpha,\delta) P_r(m) \tag{15a}$$

where f is real, known at $\left\{(\alpha_j,\delta_j;\, m_k)\right\}$ $j = 1,2,3,\ldots,J;\ k = 1,2,3, \ldots,K$. In the case of a continuous distribution

$$f_{pqr} = \int_{-1}^{+1} dm \int_{0}^{2\pi} d\alpha \int_{-\pi/2}^{+\pi/2} f(\alpha,\delta;\, m) P_r(m). \tag{15b}$$

$$Y^*_{pq}(\alpha,\delta)\cos\delta d\delta$$

When the data is sparse an alternative to this is to minimize

$$S_{PR} = \sum_{j,k}^{J,K} \left| f(j;k) - \sum_{r,p,q}^{R,P} f_{pqr} Y_{pq}(j) P_r(k) \right|^2 \tag{16a}$$

where $f(j;k) \equiv f(\alpha_j, \delta_j; m_k)$ etc. When f is real we have $f_{pqr} = (-1)^p f^*_{p-qr}$ so that all of the unknowns aren't independent. When S_{PR} is rewritten solely in terms of independent expansion coefficients, the gradient of S_{PR} computed, and then set equal to zero one derives the normal equations. They turn out to be

$$\sum_{r=o}^{R} \left\{ \sum_{p=o}^{P} \left[2 \sum_{q=+1}^{+p} \left((x_{pqr} \lambda^{pqr}_{\ell mn} - y_{pqr} \nu^{pqr}_{\ell mn} \right) \right. \right.$$

$$\left. \left. + x_{por} \lambda^{por}_{\ell mn} \right] \right\} = \sum_{j,k} f(j;k) C_{\ell m}(j) P_n(k) \tag{16b}$$

$$\sum_{r=o}^{R} \left\{ \sum_{p=o}^{P} \left[2 \sum_{q=+1}^{+p} \left((x_{pqr} \nu^{pqr}_{\ell mn} - y_{pqr} \mu^{pqr}_{\ell mn} \right) \right. \right.$$

$$\left. \left. + x_{por} \nu^{\ell mn}_{por} \right] \right\} = \sum_{j,k} f(j;k) S_{\ell m}(j) P_n(k)$$

where $f_{pqr} = x_{pqr} + iy_{pqr}$, $Y_{pq}(j) = C_{pq}(j) + iS_{pq}(j)$ (x,y,C,S real) and

$$\lambda^{\ell mn}_{pqr} = \sum_{j,k} C_{pq}(j) P_r(k) C_{\ell m}(j) P_n(k)$$

$$\mu^{\ell mn}_{pqr} = \sum_{j,k} S_{pq}(j) P_r(k) S_{\ell m}(j) P_n(k) \tag{16c}$$

$$\nu^{\ell mn}_{pqr} = \sum_{j,k} C_{pq}(j) P_r(k) S_{\ell m}(j) P_n(k)$$

IV. RESULTS

In order to decide which of the (say) $\Delta\delta_{mn}^{ns}$ [cf. Eq. (18)] are important we have used the F-test. Kendall and Stuart[11] contains a readable discussion of the topic of discarding variables including the fact that there is no purely logical way to do so. We have presumed that the lower index expansion coefficients are a priori more probably significant than are the higher index ones. In every case the 90% level of significance has been used. Moreover because the systematic differences are real the expansion coefficients satisfy

$$\Delta\delta_{mn}^{ns} = (-1)^m\left(\Delta\delta_{m-n}^{ns}\right)^*$$

A. Positions

Table 1 lists those expansion coefficients for $\Delta\alpha\cos\delta$, $\Delta\delta$, and Δp which were significant at the 90% level when each of these quantities was expanded in the form (5).

B. Proper Motions

Table 2 is similar to Table 1 but for $\Delta\mu_\alpha\cos\delta$, $\Delta\mu_\delta$, and $\Delta\mu$.

TABLE 1

EXPANSION COEFFICIENTS FOR POSITIONS

$\Delta\alpha\cos\delta$ ($0^s.001$)

n	m	Coefficient
0	0	-0.808
1	0	-1.47
2	0	8.17
3	0	-5.86
3	2	3.71 -2.08i
4	0	2.74
5	0	-3.52
7	0	-1.83
8	0	6.15
10	0	3.55

$\Delta\delta$ ($0''.01$)

0	0	16.8
2	0	12.9
3	0	-1.47
3	1	-2.29 -6.38i
4	0	12.7
4	1	-3.34 +2.56i
5	0	2.57
5	1	-3.65 +3.91i
6	0	10.5

Δp ($0''.01$)

0	0	22.8
2	0	0.713
4	0	9.78
8	0	-7.25

TABLE 2

EXPANSION COEFFICIENTS FOR PROPER MOTIONS

$\Delta\mu_{\alpha}\cos\delta$ ($0\overset{s}{.}001$ cent)

n	m	Coefficient
0	0	-5.17
2	0	20.9
3	2	4.07 -8.22i
4	0	18.7
5	2	5.40 -10.4i
6	0	-13.3
7	0	-8.10
9	0	3.34
10	0	8.89

$\Delta\mu_{\delta}$ ($0\overset{''}{.}01$/cent)

0	0	2.38
1	0	-5.13
1	1	3.72 +9.81i
2	0	22.6
2	1	5.54 -14.0i
3	0	13.2
4	0	26.1
5	0	17.2
6	0	27.0
9	0	28.2
12	0	25.3
13	0	-23.4

$\Delta\mu$ ($0\overset{''}{.}01$/cent)

0	0	7.03
4	0	11.6

C. Magnitude and Color Effects

Tables 3 and 4 list, in the same format as Tables 1 and 2, the coefficients significant at the 90% level when either magnitude or color effects are separately included for $\Delta\alpha\cos\delta$ and $\Delta\delta$ as in (15a).

TABLE 3

EXPANSION COEFFICIENTS FOR POSITIONS WITH MAGNITUDE TERMS

$\Delta\alpha\cos\delta$ ($0^{s}.001$)

n	m	r	Coefficient
0	0	0	-1.25
1	1	0	1.51 +1.17i
2	0	2	-3.84
2	2	0	6.96 +1.83i
3	0	0	-8.83
3	2	0	4.76 -3.42i
4	0	0	3.70
4	2	0	-2.60 +1.24i
5	0	0	-4.40
5	1	0	0.770 -5.17i
6	0	0	-1.10
6	0	1	-4.27
7	0	0	-1.93
7	1	0	-2.10 +2.96i
8	0	0	7.94
9	0	0	1.74

$\Delta\delta$ ($0''.01$)

n	m	r	Coefficient
0	0	0	22.0
1	0	1	14.6
1	1	0	2.36 +1.45i
2	0	0	14.7
3	0	0	-3.82
3	1	0	-2.51 -9.07i
4	0	0	15.6
4	1	0	-5.12 +2.59i
5	0	1	7.29
5	1	0	-5.24 +6.01i
6	0	0	12.3
6	1	0	4.15 -6.84i
7	0	1	-4.67
8	0	0	-11.6
9	0	0	20.2
11	0	0	-7.47
12	0	0	6.83
13	1	0	5.85 -0.879i
15	1	0	-3.29 +0.918i

TABLE 4

EXPANSION COEFFICIENTS FOR POSITIONS WITH COLOR TERMS

$\Delta\alpha\cos\delta$ ($0\overset{s}{.}001$)

n	m	r	Coefficient
0	0	0	0.495
1	0	0	3.21
1	1	0	4.26 +2.69i
2	0	0	5.95
2	1	0	-0.235 +3.98i
3	0	0	4.12
4	0	0	-1.78
5	0	0	-6.47
5	1	0	0.985 -3.21i
6	0	0	-6.39
6	3	0	-2.20 +3.69i
7	0	0	-2.23
7	1	0	-2.25 +3.32i
8	0	0	3.14
9	0	0	6.27
10	0	0	5.46
12	0	0	3.12

$\Delta\delta$ ($0\overset{''}{.}01$)

n	m	r	Coefficient
0	0	0	10.2
1	0	0	19.6
1	1	0	4.34 +2.11i
2	0	0	14.0
2	1	0	2.78 -5.55i
2	2	0	-3.80 -9.21i
3	0	0	2.47
3	0	1	-3.23
4	0	0	10.4
4	1	0	-6.26 +2.84i
5	0	0	20.0
5	0	1	6.25
6	0	0	6.38
7	0	0	-4.08
7	1	0	2.22 -3.16i
8	0	0	6.73
9	0	0	11.3
10	0	0	2.45
10	1	0	-4.72 -1.05i
13	0	0	-4.94
13	1	0	5.13 -0.858i

APPENDIX I: COMPUTING $P_{nm}(x)$

As mentioned in the Introduction we've used the University of Rochester Computing Center subroutine LEGPOL to compute $P_{nm}(x)$. We had to modify it to adequately cover the n = m case. It's also inaccurate for very small x because it raises x to the n - m power which can easily result in underflows. Several thousand values of $P_{nm}(x)$ were generated for n = 0, 1, 2,....15; m = -n, -n + 1,...n and compared to the values in reference 8. The largest discrepancy we found was 1 unit in the last decimal plaee (i.e., <5 parts in 10^8).

The argument list for LEGPOL is N, M, X, ANS, AGAMA. Here N = n, M = m, X = x, ANS = $P_{nm}(x)$, and AGAMA is a vector of the natural logarithm of the gamma function; AGAMA(I) = $\ell n[\Gamma(I+1)]$. This is used for normalization purposes. The full FORTRAN source code follows.

```
      SUBROUTINE LEGPOL(N,M,X,ANS,AGAMA)
      IMPLICIT REAL*8 (A-H,O-Z)
      DIMENSION AGAMA(31)
      IF(M.NE.0)GO TO 60
      IF(N.NE.0)GO TO 60
   63 ANS=1.D0
   20 RETURN
   60 IF(X.NE.0)GO TO 62
      IF(M.NE.1)GO TO 62
      IF(N.GT.1)GO TO 62
      GO TO 63
   62 TERM=1.D0
      K=1
      A=0
      LI=2*N-1
      DO 1 I=1,LI,2
      FNUM=I
      IF((K-M).GT.0)GO TO 3
    2 FDIV=1.D0
      GO TO 10
    3 A=A+1.D0
      FDIV=A
   10 TERM=TERM*FNUM/FDIV
    1 K=K+1
      IF(X.EQ.0)GO TO 5
      IF(N-M-1)41,65,40
   65 SUM=X
      GO TO 42
   41 SUM=1.D0
      GO TO 42
   40 I=N-M
      LI=I/2
   23 FNUM=2*N-1
      FDIV=0
      FI=I
      COV=1.D0
      A=X**I
      SUM=A
      DO 22 K=1,LI
      COV=-COV
      PRE=FI*(FI-1.D0)/((FDIV+2.D0)*FNUM*X**2)
      A=PRE*A
      SUM=SUM+A*COV
      FI=FI-2.D0
      FDIV=FDIV+2.D0
   22 FNUM=FNUM-2.D0
   42 I=M/2
      K=2*I
      A=1.D0-X*X
      FI=1.D0
      IF(M-1)77,78,79
   78 GO TO 71
   77 GO TO 70
   79 DO 75 LI=1,I
   75 FI=FI*A
      IF((M-K).NE.0)GO TO 71
   70 ANS=TERM*SUM*FI
      GO TO 20
   71 ANS=TERM*SUM*FI*DSQRT(A)
      GO TO 20
```

```
    5 IDIFF=N-M
      IF (IDIFF .EQ. 0) GO TO 53
      ID2=IDIFF/2
      IF((IDIFF-2*ID2).NE.0) GO TO 52
      XNUM=1.D0
      XDEN=1.D0
      DO 50 I=1,IDIFF
   50 XNUM=XNUM*(IDIFF-I+1)
      DO 51 I=1,ID2
   51 XDEN=XDEN*2.*I*(2.*N-2.*I+1)
      ANS=TERM*XNUM/XDEN
      IF(MOD(IDIFF,4).NE.0)ANS=-ANS
      GO TO 20
   52 ANS=0.
      GO TO 20
   53 XN=N
      ANS=DEXP(AGAMA(2*N+1)-AGAMA(N+1))*(1.D0/2.D0)**XN
      GO TO 20
      END
```

APPENDIX II: THE TIME DEPENDENCE OF <AGK3-SAOC>

The table gives the average values, for those stars in both catalogs with matching BD numbers and with the indicated Δp restrictions, the values of Δp and its dispersion (for the mean equator and equinox of 1950.0) for the epochs of place 1950.0, 1975.0, and 2000.0. For $\Delta p \leq 10''$ the means of the components at the three epochs are given too. Obviously there is a relentless secular increase in all of these quantities. In the $\Delta p \leq 2''.5$ bin the average slope is $1''.5$/cent - not a very good state of affairs.

TABLE A1

THE TIME DEPENDENCE OF <AGK3-SAOC>

	$\Delta p \leq 2''.5$		$\Delta p \leq 5''.0$	
Epoch	$\langle \Delta p \rangle$	$\sigma_{\Delta p}$	$\langle \Delta p \rangle$	$\sigma_{\Delta p}$
	(0"001)		(0"001)	
1950.0	469	296	473	316
1975.0	897	515	937	585
2000.0	1206	592	1423	849

$\Delta p \leq 10''.0$

Epoch	$\langle \Delta\alpha\cos\delta \rangle$	$\langle \Delta\delta \rangle$	$\langle \Delta p \rangle$	$\sigma_{\Delta p}$
		(0"001)		
1950.0	6	42	483	419
1975.0	9	54	948	649
2000.0	11	66	1448	919

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APPENDIX III: ASTROPHYSICS AND THE MAGNITUDE AND COLOR EFFECTS

Table A2 repeats a small part of the information contained in Tables 10 and 11 of reference 1. The questions are "Is the trend reversal in $\Delta\alpha\cos\delta$ in the last m and c bins meaningful?" and "Is the discontinuity in $\Delta\delta$ in the last m and c bins meaningful?" We argue in posing these questions that $\Delta\delta$ for $8^{m}.0 \leq m_{pg} \leq 10^{m}.5$ (the m_{pg} values are from the AGK3 and presumably homogeneous) and for $-0^{m}.5 \leq c \leq 1^{m}.5$ ($c = m_v - m_{pg}$ both apparent magnitudes from the SAOC) are constants and that both $\partial(\Delta\alpha\cos\delta)/\partial m$ and $\partial(\Delta\alpha\cos\delta)/\partial c$ are negative. The constancy of $\Delta\delta$ is beyond statistical reproach, the fact that $\partial(\Delta\alpha\cos\delta)/\partial m<0$ nearly so, but that $\partial(\Delta\alpha\cos\delta)/\partial c<0$ questionable.

Before attempting to answer these queries we observe that $c > 1^{m}$ implies that the star is red. Red stars are intrinsically faint. From Allen[9] a B-V ($\approx m_v - m_{pg}$) of $1^{m}.5$ implies $M_V = 9^{m}.8$ (M_V = absolute visual magnitude = m_v at a distance of 10pc with no absorption) for the main sequence. B-V = $1^{m}.6$ means $M_V = 11^{m}.8$. These stars, therefore, can have an $m_v \sim 10^{m}$ if and only if they are close, at about 10pc. Therefore there should be a very close correlation between the $c > 1^{m}.5$ and $m_{pg} > 10^{m}$ results. This is exactly what is seen. Moreover these stars must be distributed all over the celestial sphere (because they're close) and because if they weren't we'd know about it (e.g. Gould's Belt for the blue O and B stars). This means that they can't all (or even a

sizeable fraction) to be from one of the source catalogs of the SAOC (except for the AGK2). Hence, we've really discovered a systematic difference between the AGK2 and AGK3. We now turn to potential astrophysical causes or explanations.

Because these stars are close they'll have larger secular parallaxes than does the average star. But even if the mean epoch difference between the AGK3 and the SAOC (read AGK2) is fifty years this amounts to a milliparsec. It's not a systematic difference in the plate reduction schemes, as long as they were the same (to first order) because this is a catalog to catalog comparison. Although the presence of a color and magnitude terms ($\sim 0\overset{''}{.}25|m_{pg} - 9\overset{m}{.}12|$, see Eichhorn[10]) in the original version of the AGK2 is known, the AGK2 plate measurements were re-reduced to provide AGK3 proper motions. Similarly we rule out other plate reduction difficulties. This exhausts the astrophysical causes that immediately come to mind. Another point is the smallness of the numbers in Table A2. No position is given to better than $0\overset{''}{.}01$ and very, very few are truly known this accurately. Hence, could not Table A2 be showing us truncation effects? No, not in both right ascension and declination only for faint, red stars.

TABLE A2

THE MAGNITUDE AND COLOR EFFECTS*

m Range			$\Delta\alpha\cos\delta$	$\Delta\delta$ (0".001)	Δp	Number
$8^m_{\cdot}0$	:	$9^m_{\cdot}0$	25	38	457	25736
9.0	:	9.5	4	40	430	23247
9.5	:	10.0	-10	35	444	25576
10.0	:	10.5	-17	40	459	20800
10.5	:	11.5	-2	57	509	15429
All			3	40	465	126630
c Range						
$-0^m_{\cdot}5$	:	$0^m_{\cdot}5$	8	31	437	19821
0.5	:	1.0	-2	25	440	24233
1.0	:	1.5	-5	39	454	21746
1.5	:	2.5	12	82	499	12069

*Stars in both catalogs, with matching BD numbers, north of the equator, and with $\Delta p \leq 2".5$.

APPENDIX IV: STARS WITH LARGE POSITIONAL DIFFERENCES

Table 2 of reference 1 lists several hundred stars for which Δp was in excess of 6". We have checked each of these stars individually by looking them up in the SAOC source catalog. For the huge majority there is no large discrepancy in position. Rather these stars are members of a binary system, each star having the same DM number, with the AGK3 containing both components while the SAOC only has one. As we sorted by increasing right ascension (and clearly it would've mattered how or if we sorted) within common DM numbers and used DM number as an identifier, nearly half of all such occurrences would result in a misidentification. The majority of the remaining large Δp values apparently belong to variable stars. A few, probably less than 10, represent actual catalog errors (e. g., the SAOC right ascension for SAOC 92316 = GC 3456 = BD +15 500 is wrong). These are still being sorted out.

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SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER ESD-TR-81-286	2. GOVT ACCESSION NO. AD-A109	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle) A Comparison of the SAOC with the AGK3 Hemispherical Expansions		5. TYPE OF REPORT & PERIOD COVERED Technical Report
		6. PERFORMING ORG. REPORT NUMBER Technical Report 581
7. AUTHOR(s) Laurence G. Taff		8. CONTRACT OR GRANT NUMBER(s) F19628-80-C-0002
9. PERFORMING ORGANIZATION NAME AND ADDRESS Lincoln Laboratory, M.I.T. P.O. Box 73 Lexington, MA 02173		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS Program Element Nos. 63428F and 12424F, Proj. Nos. 3221, 2698 & 2295
11. CONTROLLING OFFICE NAME AND ADDRESS Air Force Systems Command, USAF Andrews AFB Washington, DC 20331		12. REPORT DATE 21 October 1981
		13. NUMBER OF PAGES 40
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office) Electronic Systems Division Hanscom AFB Bedford, MA 01731		15. SECURITY CLASS. (of this report) Unclassified
		15a. DECLASSIFICATION DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

Approved for public release; distribution unlimited.

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

None

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

astrometry
catalog-catalog comparison
SAOC
AGK3
position apparent magnitude, and color index effects
spherical harmonic expansions
systematic differences

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

This is the second report on the systematic differences between the AGK3 and SAOC positions and proper motions. The general theory of formulating such differences, in three different circumstances, is developed and the method of (hemi) spherical harmonics applied. Magnitude and color terms are treated, through terms quadratic in m and c, by extending the variable space via tensor product spaces. The statistical significance of an expansion coefficient is ascertained by the F test. Also included is a discussion of the time dependence of these differences and a resolution of most of the poorly matching stars (Table 2 of Part I).

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